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Mean reversion in real exchange rates

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Abstract

A modified Dickey–Fuller test, which is approximately uniformly most power invariant, is employed to examine mean reversion in real exchange rates. Results from this test provide a wider and more significant support for mean reversion than the standard Dickey–Fuller test.

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1. Introduction

The purchasing power parity (PPP) theory suggests that currencies are valued for the goods they can purchase and, in arbitrage equilibrium, the exchange rate between two countries' currencies should equal the ratio of their price levels. A testable implication is that real exchange rates should display mean reversion, at least in the long run. In contrast, an alternative theory, sometimes referred to as the ex ante PPP theory, suggests that real exchange rates may follow a martingale process with no mean reversion. Adler and Lehmann (1983), Darby (1983), Hakkio (1986), and Roll (1979), for example, note that the real exchange rate can or should follow a random walk, theoretically. The random walk proposition is based on a number of empirically questionable assumptions, namely interest rate parity holds, the forward rate is an unbiased predictor of the future spot rate, the Fisher relationship holds, and real interest rates are constant. The proposition seems difficult to be rejected empirically, however. Adler and Lehmann (1983), Frenkel (1981), and Roll (1979) report that the real exchange rate follows closely a random walk, suggesting little tendency for deviations from PPP to reverse. Huizinga (1987) presents some evidence of mean reversion in the real exchange rate, but not statistically significant. Using cointegration analysis, Baillie and Selover (1987), Corbae and Ouliaris (1988), Mark (1990), and Taylor (1988) also fail to find a long-run relationship between exchange rates and relative prices.

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Hakkio (1986) observes that the empirical failure to find mean reversion in real exchange rates can be due to the low power of standard unit-root tests. Mishkin (1984) and Cumby and Obstfeld (1984) report that ex ante PPP can be rejected when a joint test with some other proposition is considered. More recently, Abuaf and Jorion (1990) address the low-power problem by estimating unit-root regression equations jointly and they find mean reversion in real exchange rates. Diebold et al. (1991) employ fractional unit-root analysis and report evidence of mean reversion in real exchange rates under the gold standard. The Diebold et al. analysis can raise the power of the test for mean reversion by allowing for a wider range of mean reversion than standard unit-root tests (see also Cheung and Lai, 1993). All these results appear to support that standard unit-root tests have low power to detect mean reversion.

This paper further examines the mean-reverting behavior of real exchange rates by using a new unit-root test devised by Elliot et al. (1992). Their study analytically derives the asymptotic power envelope for unit-root tests. They propose a simple modification of the augmented Dickey–Fuller (1979) test such that the modified test, called the DF–GLS$^{\tau}$ test, can nearly achieve the power envelope. The DF–GLS$^{\tau}$ test is also shown to be approximately uniformly most power invariant. Monte Carlo results reported by Elliot et al. (1992) indicate that the power improvement of the modified Dickey–Fuller test can be quite large and that it exhibits good size and size-adjusted power properties in finite samples with dependent errors.

The paper is organized as follows. Section 2 describes the data. Section 3 discusses the statistical procedure used, and Section 4 contains the empirical results. Section 5 concludes.

2. The data

The data under study are annual real exchange rate data constructed from nominal exchange rates and national price levels measured by both consumer prices indexes (CPIs) and wholesale price indexes (WPIs). Data from nine countries are considered, and they include Canada, France, Germany, Italy, Japan, Netherlands, Switzerland, the United Kingdom, and the United States. The data sample covers the period from 1900 through 1992, except for the WPI series for Switzerland, which is available from 1914 to 1992. Data on annual exchange rates and WPIs for the period 1900–1972 are taken from Lee (1978), whereas the annual CPIs from 1900 to 1989 are drawn from Maddison (1991). These three sets of data series are updated and extended to 1992 using annual data taken from the International Monetary Fund's *International Financial Statistics* data tape. There are in total 16 real exchange rate series against the U.S. dollar: eight series of CPI-based real exchange rates and eight other series for WPI-based real exchange rates. Following the practice in previous studies, all data series under examination are expressed in natural logarithms.

3. A modified Dickey–Fuller test

In testing for a unit root in an autoregressive (AR) model that allows for a linear time trend, Elliot et al. (1992) show that a modified Dickey–Fuller test, referred to as the DF–GLS$^{\tau}$ test, is approximately uniformly most power invariant (UMPI) while strictly no UMPI test exists.

These authors explicitly derive the asymptotic power envelope by analyzing the sequence of Neyman–Pearson tests of the unit-root null hypothesis ($\alpha = 1$) against the local alternative of $\bar{\alpha} = 1 + \bar{c}/T$ in the finite-sample Gaussian AR($p + 1$) model, in which T is the sample size and $\bar{c}$ is some fixed constant. On the basis of the asymptotic power calculation, Elliot et al. (1992) establish that the DF–GLS$^{\tau}$ test can achieve a substantial gain in power over conventional unit-root tests. In addition, the DF–GLS$^{\tau}$ test displays good power and little size distortion in finite samples with dependent errors.

Let $\{y_t\}$ be the data process under examination. The DF–GLS$^{\tau}$ test is carried out based on the following regression:

$$(1 - \mathrm{L})y_t^{\tau} = a_0 y_{t-1}^{\tau} + \sum_{j=1}^{p} a_j (1 - \mathrm{L})y_{t-j}^{\tau} + \text{error}, \tag{1}$$

where L is the usual lag operator and y_t^{τ}, the locally detrended data process under the local alternative of $\bar{\alpha}$, is given by

$$y_t^{\tau} = y_t - \tilde{\beta}' z_t, \tag{2}$$

with $z_t = (1, t)'$ and $\tilde{\beta}$ being the regression coefficient of $\tilde{y}_t$ on $\tilde{z}_t$, for which $(\tilde{y}_1, \tilde{y}_2, \ldots, \tilde{y}_T) = (y_1, (1 - \bar{\alpha}\mathrm{L})y_2, \ldots, (1 - \bar{\alpha}\mathrm{L})y_T)$ and $(\tilde{z}_1, \tilde{z}_2, \ldots, \tilde{z}_T) = (z_1, (1 - \bar{\alpha}\mathrm{L})z_2, \ldots, (1 - \bar{\alpha}\mathrm{L})z_T)$. The DF–GLS$^{\tau}$ test statistic is given by the usual t-statistic testing $a_0 = 0$ against the alternative of $a_0 < 0$ in regression (1). Elliot et al. (1992) recommend that the parameter $\bar{c}$, which is responsible for defining the local alternative through $\bar{\alpha} = 1 + \bar{c}/T$, be set equal to -13.5. Critical values for the DF–GLS$^{\tau}$ test statistic are provided by Elliot et al. (1992, Table 1) using the Monte Carlo method.

4. Empirical results

To illustrate the possible difference in test results between the standard Dickey–Fuller test and the modified Dickey–Fuller test, all the series of real exchange rates are each first checked for mean reversion using the augmented Dickey–Fuller (ADF) test with a linear time trend. The statistical results of the ADF test are reported in Table 1. The lag length used for the ADF test is determined using a model selection procedure based on both the Akaike information criterion (AIC) and the Schwarz information criterion (SIC). For the CPI-based real exchange rate series, in only two cases (France and Germany) out of the eight cases under consideration can we find significant evidence of mean reversion. Similar results apply to the WPI-based real exchange rate series, for which the null of a non-stationary process can be rejected in just two cases (France and Japan). Hence, in accordance with previous findings, the ADF test finds little evidence of mean reversion in real exchange rates.

Table 2 contains the statistical results for the DF–GLS$^{\tau}$ test, which has been shown by Elliot et al. (1992) to be more powerful than the ADF test. The lag order used for this test is again selected based on both the AIC and SIC. For the CPI-based real exchange rate series, significant evidence in favor of mean reversion can be obtained in five out of eight cases at the 5% level or better. For the WPI-based real exchange rate series, evidence of mean reversion

Table 1
Results of the standard augmented Dickey–Fuller test

Country series	AIC lag	ADF statistic	SIC lag	ADF statistic
(a) *CPI-based real exchange rates*				
Canada	1	−2.9790	1	−2.9790
France	4	−4.3012**	1	−3.6633*
Germany	1	−11.6907**	1	−11.6907**
Italy	1	−3.4397	1	−3.4397
Japan	1	−3.1588	1	−3.1588
Netherlands	2	−2.5777	2	−2.5777
Switzerland	2	−3.0393	2	−3.0393
United Kingdom	2	−3.2140	2	−3.2140
(b) *WPI-based real exchange rates*				
Canada	4	−3.4481	4	−3.4481
France	3	−4.8396**	1	−4.4151**
Germany	1	−2.3656	1	−2.3656
Italy	1	−3.2975	1	−3.2975
Japan	2	−4.4638**	2	−4.4638**
Netherlands	1	−3.1814	1	−3.1814
Switzerland	2	−3.1271	2	−3.1271
United Kingdom	2	−3.3205	2	−3.3205

Notes: For $T = 100$, the critical values for the ADF test at the 5% and 1% levels are given by −3.45 and −4.04, respectively. Statistical significance is indicated by an asterisk (*) for the 5% level and a double asterisk (**) for the 1% level.

can be found in all but one case at the 5% level or better. In general, these results are apparently much more supportive of mean reversion in real exchange rates than those obtained based on the usual ADF test.

5. Conclusion

While the PPP theory implies the presence of mean reversion in real exchange rates, a competing theory of ex ante PPP suggests that real exchange rates can, or should be, not mean-reverting. Previous analyses of the behavior of real exchange rates using standard unit-root tests have mostly failed to detect mean reversion. The empirical failure to find mean reversion in real exchange rates does not lead to an outright rejection of PPP and acceptance of ex ant PPP, but rather they may just reflect the low power of the unit-root tests applied.

This study explores the relevance of the explanation based on low statistical test power. The DF–GLS$^{\tau}$ test recently developed by Elliot et al. (1992) is employed to test for mean reversion in real exchange rates for the period 1900–1992. This modified Dickey–Fuller test can be shown to be approximately UMPI and is much more powerful than usual unit-root tests. The improvement in test power is shown to be important for a proper evaluation of the mean-reverting behavior of the real exchange rate. It is found that the modified Dickey–Fuller

Table 2
Results of the modified Dickey–Fuller test

Country series	AIC lag	DF–GLS$^{\tau}$ statistic	SIC lag	DF–GLS$^{\tau}$ statistic
(a) *CPI-based real exchange rates*				
Canada	1	−2.8836	1	−2.8836
France	4	−4.2990**	1	−3.5493*
Germany	1	−11.3828**	1	−11.3828**
Italy	1	−3.3124*	1	−3.3124*
Japan	1	−3.1263*	1	−3.1263*
Netherlands	2	−2.4531	2	−2.4531
Switzerland	2	−2.9582	2	−2.9582
United Kingdom	2	−3.1314*	2	−3.1314*
(b) *WPI-based real exchange rates*				
Canada	4	−3.4141*	4	−3.4141*
France	3	−4.7623**	1	−4.3201**
Germany	1	−2.2672	1	−2.2672
Italy	1	−3.2859*	1	−3.2859*
Japan	2	−4.3771**	2	−4.3771**
Netherlands	1	−3.1117*	1	−3.1117*
Switzerland	2	−3.0569*	2	−3.0569*
United Kingdom	2	−3.1756*	2	−3.1756*

Notes: For $T = 100$, the critical values for the DF–GLS$^{\tau}$ test at the 5% and 1% levels are given by −3.03 and −3.58, respectively (Elliot et al., 1992). Statistical significance is indicated by an asterisk (*) for the 5% level and a double asterisk (**) for the 1% level.

test with improved power can yield results rather different from those obtained based on the standard Dickey–Fuller test. In comparison with the results obtained from the standard Dickey–Fuller test, those obtained from the DF–GLS$^{\tau}$ test are shown to be considerably more favorable to the hypothesis of mean reversion, and the DF–GLS$^{\tau}$ test results provide, accordingly, a wider and more significant support for mean reversion in real exchange rates.

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